Lines on Del Pezzo surfaces and transfinite heterotic string spacetimes

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Abstract

It is pointed out that the hierarchy of fractal dimensions characterizing transfinite heterotic string spacetimes bears a striking resemblance to the sequence of the number of lines lying on Del Pezzo surfaces.

Employing the notion and properties of the so-called Cantorian fractal space, $\mathcal{E}^{(\infty)}$, El Naschie has recently demonstrated [1–5] that the transfinite heterotic spacetimes are endowed with the following five characteristic dimensions

$$D_\kappa = \frac{(\alpha_0^{-1})\phi^{2+\kappa}}{\langle d_c^{(2)} \rangle} = \frac{\bar{\alpha}_0}{2}\phi^{2+\kappa}, \tag{1}$$

where $\kappa = 0, 1, \ldots, 4$, $\bar{\alpha}_0$ is the inverse value of the fine structure constant and ϕ stands for the Hausdorff dimension of the zero Cantor set. Taking $\bar{\alpha}_0 = 137$ and $\phi = 0.618034$, he found the following remarkable series [1]

$$D_0 \simeq 26 \;\rightarrow\; D_1 \simeq 16 \;\rightarrow\; D_2 \simeq 10 \;\rightarrow\; D_3 \simeq 6 \;\rightarrow\; D_4 \simeq 4. \tag{2}$$

The aim of this short contribution is to show that this sequence can almost exactly be reproduced by the arrangement of the multiplicities of the configurations of lines lying on Del Pezzo surfaces.

A Del Pezzo surface [6], F_N, is a normal surface of order N sitting in an N-dimensional projective space, where N=3,4,...,9. Taken together, these surfaces form a single simple series $F_9 \rightarrow F_8 \rightarrow \ldots \rightarrow F_3$, such that F_N $(3 \leq N \leq 8)$ is always the projection of F_{N+1} from a point of itself. In addition, all of them are represented on a projective plane by means of systems of non-singular cubic curves having $9 - N$ points in common – the latter being usually referred to as the base points. A line of F_N is mapped on the plane into a base point, a line joining two base points, or a conic passing through five of the base points (see, e.g. [7]); hence, the number of lines lying on F_N, $\Theta(N)$, is simply

$$\Theta(N) = (9 - N) + \binom{9-N}{2} + \binom{9-N}{5}, \tag{3}$$

with the understanding that $\binom{a}{b} \equiv 0$ if $a < b$. In particular, for $3 \leq N \leq 7$ we have

$$\Theta(3) = 27 \;\rightarrow\; \Theta(4) = 16 \;\rightarrow\; \Theta(5) = 10 \;\rightarrow\; \Theta(6) = 6 \;\rightarrow\; \Theta(7) = 3, \tag{4}$$

which, indeed, parallels so closely the sequence given by Eq. (2). It is worth noticing that

$$\Theta(3) - \Theta(4) = 27 - 16 = 11 = 10 + 1 = \Theta(5) + 1 \tag{5}$$

and

$$\Theta(5) - \Theta(6) = 10 - 6 = 4 = 3 + 1 = \Theta(7) + 1. \tag{6}$$

In order to better understand the origin of this hierarchy, as well as to see how intricate the connection between the individual Del Pezzo surfaces is, we project F_N, $N \geq 4$, into a three-dimensional projective space, denoting these projected surfaces as $\widehat{F}_N$. We first take our familar cubic surface $F_3 \equiv \widehat{F}_3$, and the twenty-seven lines on it [8]. If we disregard any one line and the ten lines which are incident with it, then the *sixteen* remaining lines are, as for their mutual intersections, related to each other as the sixteen lines lying on $\widehat{F}_4$. Analogously, if on $\widehat{F}_4$ we ignore any one line and the five lines that meet it, the *ten* remaining lines have the same intersection properties as the ten lines on $\widehat{F}_5$. Similarly, if on $\widehat{F}_5$ we omit one line and the three lines incident with it, we are left with *six* lines exhibiting the same algebra as the six lines situated on $\widehat{F}_6$. And finally, if on $\widehat{F}_6$ we leave out any one line and the two lines that meet it, the configuration of the *three* remaining lines enjoys the same properties as that of the three lines upon $\widehat{F}_7$.

And thus, since the cubic surface F_3 is a *boundary* member of the Del Pezzo series, all the above-introduced facts and findings cast completely new light on our hypothesis put forward in [8], which allows of the following intriguing extension and generalization: *the hierarchy of characteristic dimensions exhibited by the transfinite heterotic string spacetimes has its algebraic counterpart in the relationship between the configurations of lines lying on Del Pezzo surfaces of order three to seven.*

References

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